

PREDICTION OF CUSTOMERS' LOYALTY AND MARKET SHARE  
BY USING MARKOV CHAIN MODEL

1. INTRODUCTION

For any company to *measure the change of the market share of its brand or brands, and its customers' brand loyalty are strategic issues*. The *Brand Switching Model* is a way of measuring this change. So now I would like to present a real world application of the special Markov Chain model which is called the *Brand Switching Model (BSM)*. I am going use this model to *analyse the washing powder market in Budapest*.

2. DESCRIPTION OF BRAND SWITCHING MODEL

Customers' brand switching and loyalty follows a special Markov process, that can be analysed by *Brand Switching Model*.

**It has Markov property**

Customers brand choices in a given time period depend only upon their choices in the previous time period.

$$P \{X_{t+1}=j \mid X_0 = k_0, X_1 = k_1, \dots, X_t = i\} = P \{X_{t+1}=j \mid X_t = i\} = p_{ij}$$

The  $p_{ij}$  ( $i=1,2, \dots, k; j=1,2, \dots, k$ ) shows customers' switching from brand  $i$  to brand  $j$ . It is called one-step transition probability.

**It has finite state**

The number of brands of any given product or service is finite.

**The one-step transition matrix**

$$P = \begin{bmatrix} p_{00} & p_{01} & \dots & p_{0k} \\ p_{10} & p_{11} & \dots & p_{1k} \\ \cdot & & & \\ p_{k0} & p_{k1} & \dots & p_{kk} \end{bmatrix} \quad \text{is a stochastic matrix } \sum_{i=0}^k p_{ij} = 1,$$

because  $p_{i0}, p_{i1}, \dots, p_{ik}$  are conditional probabilities of mutually exclusive and exhaustive states of the system.

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### The transition probabilities are stationary

Customers' switching from one brand (state) to another remains constant over time if the initial marketing conditions do not change.

$$p_{ij}(t) = P(X_{t+1} = j \mid X_t = i) = P(X_1 = j \mid X_0 = i) = \dots = p_{ij}$$

### The n-step transition probabilities

$p_{ij}^{(n)} = P(X_{t+n} = j \mid X_t = i)$  can be determined based on the one-step transition matrix.

$$\mathbf{P}^{(n)} = \mathbf{P}^n$$

### It has a steady or equilibrium state, that gives a long run forecast of the brand distribution

$$\lim_{n \rightarrow \infty} p_{ij}^{(n)} = p_j.$$

### It has an $\mathbf{m}_t^*$ initial state vector,

which shows the initial market share of brands. The

$$\mathbf{m}_{t+1}^* = \mathbf{m}_t^* \mathbf{P}, \mathbf{m}_{t+2}^* = \mathbf{m}_{t+1}^* \mathbf{P} = \mathbf{m}_t^* \mathbf{P}^2, \dots$$

gives short run forecasts of brand distributions.

## 3. A REAL WORLD APPLICATION OF BRAND SWITCHING MODEL

In February of year 1998 and year 2000 I assessed and analysed the washing powder market of Budapest in order to make forecasts of the market share distribution and customers' brand loyalty by using the BSM.

In Hungary the major washing powder producers and brands are in the following table:

Manufacturing Co.	Produced Brand
Procter & Gamble	Ariel, Tix
Henkel	Persil, Tomi
Unilever	Omo, Biopon
Others (Benchier, EVM)	Others (Dosia, Ultra Viola)

*My assessment of the washing powder purchasing behaviour of Budapest's households* was based on a paper and pencil survey, which was set up and carried out by *two groups of third year students studying Operations Research* at the College of Management and Business Studies, Budapest.

*The sample size* was 600 customers in each year on a given week in February. The survey was taken randomly in different supermarkets and shopping malls.

I found that:

- The quantity of washing powder used depends only upon the size of households.
- Usually quantity of washing powder per household was purchased for 4 or more weeks.
- The favourite package sizes were 5kg, 5.4kg and 8 kg.
- The quality of washing powder purchased depends mainly on the income of households.

In the centre of the BSM stands the  $\mathbf{P} = [p_{ij}]$  transition matrix, where  $p_{ij}$  means the probability that customers switch from brand  $i$  to brand  $j$  for all  $i, j$  ( $i, j = 1, 2, \dots, k$ ).

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In our case the number of brands was seven in the washing powder market in Budapest, so the type of the **P** matrix is 7 by 7 .

In order to quantify the **P** matrix my students registered *the purchased brand and quantity for each sampled customer*; and asked the *brand of the previous washing powder purchased and the frequency of purchasing*.

Our sample contains households of different sizes and different quantities of washing powder purchased.

On the basis of my findings I defined 400-500 gram washing powder usage as *standard unit per household*, and transformed each purchased quantity into multiple standard units.

In 1998 our survey concerned 921 standard units of washing powder, while in 2000 it was 964 standard units based on 600 - 600 respondents.

### 3.1. Observation of the change in market share and customers' loyalty according to brands

The results of surveys in 1998 was the following:

*Table 1.*  
*1998 - Transition probabilities according to brands*

Brand ↓	→	Ariel	Tix	Persil	Tomi	Omo	Biopon	Other	Σ
Ariel		0.625	0.0	0.250	0.058	0.0	0.038	0.029	1.000
Tix		0.051	0.385	0.026	0.290	0.026	0.077	0.145	1.000
Persil		0.250	0.121	0.345	0.131	0.036	0.071	0.046	1.000
Tomi		0.066	0.147	0.044	0.534	0.015	0.162	0.032	1.000
Omo		0.200	0.044	0.244	0.156	0.300	0.0	0.056	1.000
Biopon		0.137	0.137	0.0	0.296	0.0	0.389	0.041	1.000
Other		0.073	0.228	0.049	0.160	0.086	0.080	0.324	1.000

In the *main diagonal* the probabilities of brand loyalties (**holding power**) can be find.

The *rows* show the probabilities switching (**pushing power**) from given brand *i* to *j*.

The *columns* show the probabilities switching (**attraction power**) from brand *j*-s to given brand *i*.

Initial state vector:

$$\mathbf{m}_t^* = [0.234; 0.138; 0.153; 0.258; 0.036; 0.106; 0.075]$$

shows the washing powder market share in 1998 according to brands.

Let's look at the result of the solved model.

The short run forecasts in 1998:

$$\mathbf{m}_{t+1}^* = [0.236; 0.143; 0.139; 0.260; 0.030; 0.119; 0.073]$$

$$\mathbf{m}_{t+2}^* = [0.234; 0.144; 0.133; 0.264; 0.028; 0.124; 0.072]$$

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Based on the BSM my expectation in the short run was that the market share of *Ariel*, *Persil* and *Omo* would slightly decrease, and the position of *Tomi* and *Biopon* would slightly improve.

If initial marketing conditions were invariable on the market in the future, in the long run we would expect that the washing powder market reach the following steady state:

$$\mathbf{m}_{t+19}^* = [0.227; 0.147; 0.126; 0.271; 0.027; 0.129; 0.073],$$

and in February 2000 our survey would have to show this market share.

The result of surveys in 2000 is the following:

*Table 2.*  
*2000 - Transition probabilities according to brands*

Brand → ↓	Ariel	Tix	Persil	Tomi	Omo	Biopon	Other	Σ
Ariel	0.665	0.070	0.070	0.090	0.018	0.045	0.042	1.00
Tix	0.099	0.436	0.035	0.205	0.057	0.130	0.038	1.00
Persil	0.180	0.012	0.453	0.176	0.041	0.015	0.015	1.00
Tomi	0.138	0.136	0.048	0.632	0	0.019	0.027	1.00
Omo	0.271	0.163	0.154	0.041	0.271	0.100	0	1.00
Biopon	0.147	0.237	0.058	0.189	0.084	0.227	0.158	1.00
Other	0.136	0.096	0.223	0.175	0.051	0.020	0.299	1.00

*The initial marketing conditions changed during the observed time period – modified brands appeared on the market, promotional campaigns, discount prices and so on affected the behaviour of customers.*

Due to these, *the initial market share*

$$\mathbf{m}_t^* = [0.269; 0.171; 0.115; 0.277; 0.046; 0.065; 0.057]$$

*in 2000 slightly differs from the forecasted one.*

For example, the position of *Ariel* strengthened. My findings showed its market share and customers' brand loyalty increased because of TV ads, the introduction of a new improved brand, and price discounts.

The market share of *Ariel* increased from 23.4% to 26.9%, and it increased its brand loyalty index as well.

Let us compare the brand loyalty in the washing powder market from year 1998 to year 2000.

*Table 3.*

Year	Ariel	Tix	Persil	Tomi	Omo	Biopon	Other
1998	62.5	38.5	34.5	53.4	30.0	38.9	32.4
2000	66.5	43.6	45.3	63.2	27.1	22.7	29.9
Change	+4%	+5.1%	+10.8	+9.8	-2.9	-16.2	-2.5

The brand loyalty of *Persil* and *Tomi* increased by about 10%, and the brand loyalty of *Biopon* decreased by 16.2%.

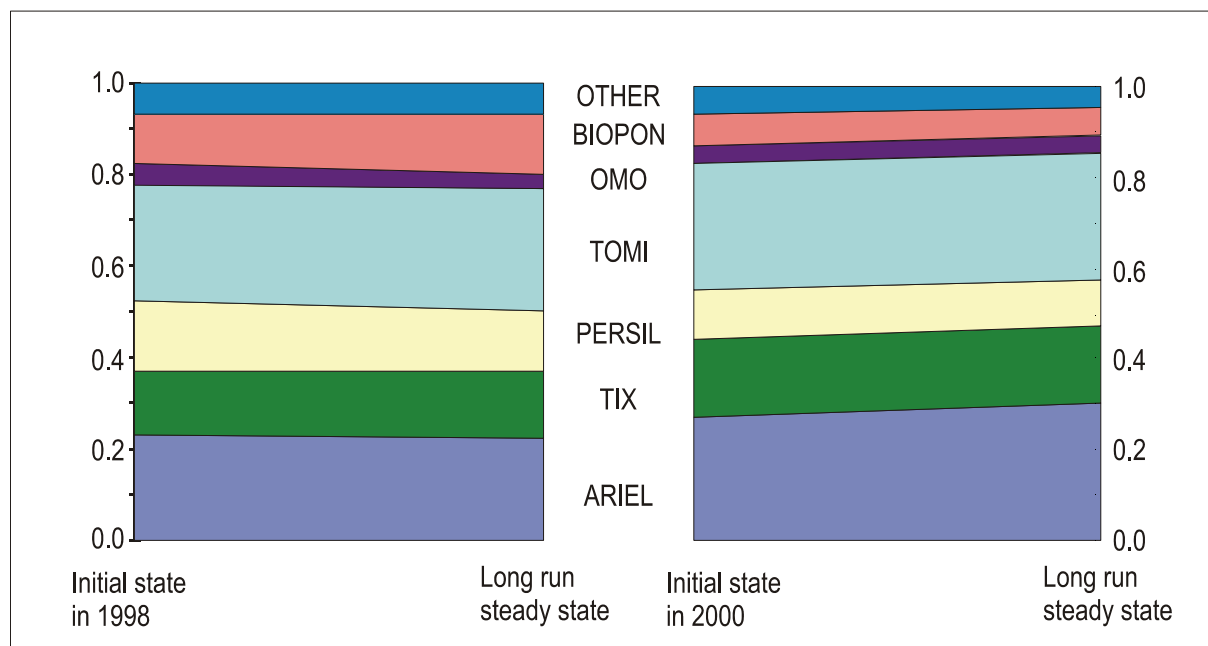
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The steady state based on Brand Switching Model is the following:

$$\mathbf{m}_{t+14}^* = [0,299; 0,171; 0,109; 0,278; 0,037; 0,061; 0,045].$$

Our expectation based on BSM is that the market share of Ariel will increase by a further 3% if marketing conditions on the washing powder market in Budapest remain the same.

*Figure 1.  
Brand distribution in the washing powder market in Budapest*



As it shows, the market position of Biopon worsened, there is 16% decrease in customer loyalty. This is an extremely high decrease and this should be a clear warning that there is a problem with for instance *Marketing* or *Operation Management*.

### 3.2. Observation of the change in market share and customers' loyalty according to manufacturers

*Table 4.  
1998 - Transition probabilities*

Co.	P& G	Henkel	Unilever	Other	$\Sigma$
P& G	0.560	0.311	0.060	0.069	1.000
Henkel	0.278	0.536	0.148	0.038	1.000
Unilever	0.258	0.345	0.349	0.048	1.000
Other	0.302	0.207	0.165	0.326	1.000

Initial state vector:

$$\mathbf{m}_t^* = [0.372; 0.411; 0.142; 0.075]$$

Steady state vector:

$$\mathbf{m}_{t+n}^* = [0.386; 0.398; 0.144; 0.072]$$

*Table 5.  
2000 - Transition probabilities*

Co.	P& G	Henkel	Unilever	Other	$\Sigma$
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P& G	0.661	0.189	0.109	0.041	1.000
Henkel	0.284	0.661	0.032	0.023	1.000
Unilever	0.403	0.228	0.333	0.036	1.000
Other	0.232	0.398	0.071	0.299	1.000

Initial state vector:

$$\mathbf{m}_t^* = [0.436; 0.398; 0.110; 0.057]$$

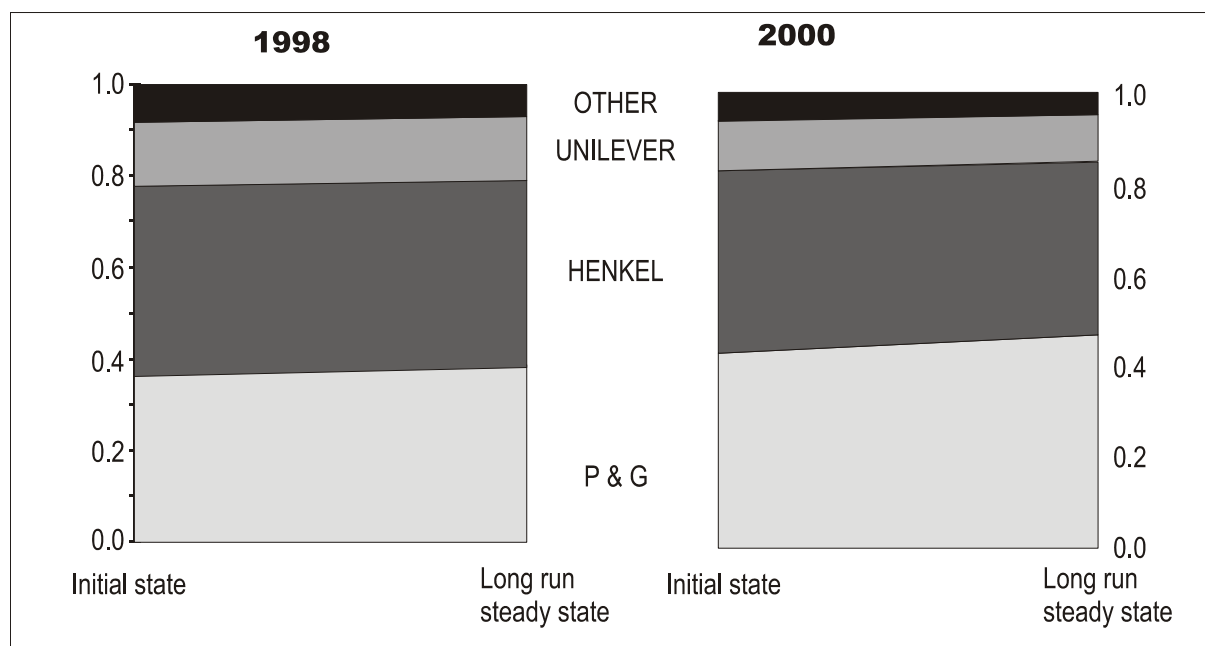
Steady state vector:

$$\mathbf{m}_{t+n}^* = [0.472; 0.384; 0.100; 0.045]$$

As the result of the given Brand Switching Model, the position of *Ariel* and its *manufacturer*, P&G is strong on the given market probably because of the applied successful marketing strategy.

The position of *Tom* and its *manufacturer*, Henkel is also good on the washing powder market of Budapest.

Figure 2.  
Brand distribution in the washing powder market in Budapest according to companies



## 4. CONCLUSION

The analysis shows that BSM could be used as an *important diagnostic tool for management decision-makers* in order to determine the appropriate effective marketing or production intervention to strengthen the market position of a given brand.

I recommend this application be used to help *forecast* the market share and brand loyalty for *competitive products and services* as well.

The *repeated application* of BSM can also be useful in measuring the effect of a new marketing strategy between two time periods.

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