1. Introduction

Organizations are generally defined as structured complexes of units that are managed for definite goals. Based on organizational theory the development of formal models for analysis of organizations, and in turn the analysis of organizational modelling using of formal models, including computational models and mathematical models is attractive topics in organizational researches (see [So]). Kathleen M. Carley in [Kath] sketched out the perspectives and directions of organizational modelling.

In general a company (an *organization)*, its *structure and its operational characteristics are inseparable factors that jointly decide the* company results. Therefore the management of a company requires thorough analysis of the operative units and departments belonging to the company as its components, the relationships between them, as well as their capacities in different activities of the company. A comprehensive study of companies with its hierarchical components, of their structure and of the capacities of components as separate factors gives advantages:

- The managers can view clearly their companies with its components, the relationships between them and the capacities of each components.

- In the company management the managers can see the roles of the components, the roles of the relationships between the components and the roles of the capacities of each components in the development of the company. The managers can see the affects resulted in by any change of the components, of their relationships or of their capacities.

In following sections the companies and systems with their components, commonly called by organizations, are examined. A company in a point of view is a hierarchical association of operative units and departments with their own functions and abilities. Therefore the companies can be considered as *structured organizations*. The system of relationships between units and departments forms the *structure* of the companies. The relationships in their nature may be independent, cooperative, sequential or hierarchical. The units and departments operate in the company with determined functions and capacities.

Based on these analysis an approach to modelling of companies with their structures is proposed below and a list of problems concerning the company's management is considered.

2. Modelling of structured organizations

Modelling of structured organizations is based on their three constituting components: operative units, structure and operational characteristics of the units.

Structured organizations in fact are complexes of connected units that:

- are combined equally or built up one upon other in some hierarchy,
- are set in definite positions in an operational network in an operational processes,
- operate with definite functions, and
- operate with definite capability in definite conditions.

One of most important factors that affects the structured organizations effectiveness is their structure. By structure we can understand the systems of relationships between the units and the divisions belonging to organizations. An organization in different economical processes may be embedded at same time into different structures. The organizational structures determine the administrative connections between units and divisions, while in material receiving-supplying processes or in information receiving-delivering processes the

organizations operate in *material distributing*, or *information delivering* structures. Each type of structures has its own features and the processes work well if the suiting is perfect.
Another factor in organization modelling is the characteristics of units and divisions. In different points of view the characteristics may be different: specialization, functionality, performance, ability and capability, etc. The *functionality* structured organizations is expressed in the ability of them in processing inputs and producing outputs. The structured organizations complete the production processes with their *performances*. The performances of superior divisions depend on the performances of constituting components. The dependency is determined by the relationships between the divisions and the constituting components. From another point of view, if the structured organizations are set into material receiving-supplying processes, or into information receiving-supplying processes, other characteristics of their units and divisions must be considered. They may be storage capability in logistical processes or information processing capability in information management.
In the following sections from different points of view some approaches are suggested to modelling structures and structured organizations.

3. Structures of organizations

The organizations are sets of functional units arranged in some structures. The structures are the frames of organizations that in a way describe the relationships between units. Therefore, the structures in general should be defined by mixed graphs (see [Hans]).
Definition 1: By a structure we mean a mixed graph $G = \langle V, E\rangle$, where V is a finite set of vertices (nodes), $E \subseteq \{[v_1, v_2]|\, v_1, v_2 \in V\} \cup \{(v_1, v_2)|\, v_1, v_2 \in V\}$ is a set of edges of G in which if $(v_1, v_2) \in E$ then we have also $(v_2, v_1) \in E$ and $(v_1, v_2) = (v_2, v_1)$.
An edge $e \in E$ of G is directed, if $e \in \{[v_1, v_2]|\, v_1, v_2 \in V\}$, otherwise it is undirected edge.
For two structures G_1, G_2 where $G_1 = \langle V_1, E_1\rangle$ and $G_2 = \langle V_2, E_2\rangle$ we say that G_1 is a sub-structure of G_2 if G_1 is a sub–graph of G_2.
One organization may operates in different structures accordingly to the purpose of the study. Typically, the frequent structures are:

- *Organizational structures*: The structures show the inner relationships between subunits in the point of view of management. On one hand the organizational structures present the establishment of the organizations, and on the other hand they show the management flows in the organizations. The organizational structures therefore cannot contain closed directed paths. The organizational structure is the basic structure of the organizations.

- *Instruction processing structures*: In the case that the organizations are considered in an organizational structure, the relation between organizations is the relation between superordinate and subordinate units and the function of these units is to fulfill the duties given by superordinate organizations. In general the instruction processing structures are acyclic. In other words the instruction processing structures can be represented by tree graphs where the directed edges between nodes denote the relationship between superordinate and subordinate units.

- *Material distributing structures*: If the organizations work in a logistic network, the material distributing structures described the relationships between the senders and the receivers. Then the main duty of the units in the system is material distribution. In general the material distribution is also acyclic and distributive process: the material distribution is executed in a tree formed network and the total of delivered items cannot exceed the total of coming items. The units in material distributing structures can store the items within a given storage limit.

- *Information delivering structures*: In the organizations the information sharing processes and information transmission processes take place in the networks of equal or different leveled units. By information transmission and information sharing we distinguish two ways of information processing: by transmission we mean the addressed information sending from superordinate units to subordinate units, while by information sharing we mean the untargeted information delivering between equal leveled units.

4. Organizational structure

The relationship between the units in a structure may be hierarchical or non-hierarchical. The hierarchical relationships are the relationships between superordinate units and subordinate units. The non-hierarchical relationships are the relationships between the units of same level. The non-hierarchical relationship may be

- *separate*: The units in the relationship operate independently.
- *cooperative*: The units in cooperative relationship operate independently each from other. Every unit in the relationship processes the input and gives its own products. The total production of the whole sub-organization is aggregated from the products of the units in the sub-organization, and the performance of the whole sub-organization is determined as the total of the performance of the units in the sub-organization.
- *sequential*: The units in sequential relationship operate in a production line in which each unit processes the previous units products. The total production of the whole sub-organization is the production of the last unit in the line, and the performance of the whole sub-organization is determined by the performance of the weakest unit in the line.

Formally the structures over A are defined recursively as in Definition 2:

Definition 2:

i. (**Atomic structures**) Every $a \in A$ can be considered as an atomic structure that at same time may be:

- a separative structure $s = (a)$. The type of s is $t(s) = (*)$.
- a cooperative structure $s = [a]$. The type of s is $t(s) = [*]$.
- a sequential structure $s = \langle a \rangle$. The type of s is $t(s) = \langle * \rangle$.

If s is an atomic structure, then the set of substructures of s and the set of atomic substructures of s is $\boldsymbol{SS}(s)$, and $\boldsymbol{AS}(s)$, respectively, and we have $\boldsymbol{SS}(s) = \{s\}$, $\boldsymbol{AS}(s) = \{a\}$.

ii. Let $s_1, s_2, \dots, s_n$ be structures over A, where the type, the set of substructures and the set of atomic substructures of s_i is t_i, $\boldsymbol{SS}(s_i)$, and $\boldsymbol{AS}(s_i)$, respectively, for $i = 1,2,\dots,n$.

- (**Separative structure**) By $s = (s_1, s_2, \dots, s_n)$ we denote the seperative structure formed by different $s_1, s_2, \dots, s_n$.
- (**Cooperative structures**) By $s = [s_1, s_2, \dots, s_n]$ we denote the cooperative structure formed by different $s_1, s_2, \dots, s_n$.
- (**Sequential structures**) By $s = \langle s_1, s_2, \dots, s_n \rangle$ we denote the sequential structure formed by $s_1, s_2, \dots, s_n$.

If s is seperative, cooperative or sequential structure formed by $s_1, s_2, \dots, s_n$ then the type of s is $t(s) = (t_1, t_2, \dots, t_n)$, $t(s) = [t_1, t_2, \dots, t_n]$ or $t(s) = \langle t_1, t_2, \dots, t_n \rangle$, respectively. The set of substructures of s is $\boldsymbol{SS}(s) = \{s\} \cup \bigcup_{i=1}^{n} \boldsymbol{SS}(s_i)$ and the set of atomic substructures of s is $\boldsymbol{AS}(s) = \bigcup_{i=1}^{n} \boldsymbol{AS}(s_i)$.

iii. All structures over A are defined as in i. and ii.

The set of all structures over A are denoted by $\boldsymbol{S_A}$.

Definition 3: Two structures r, s are of the same type if $t(r) = t(s)$.

Remark:

1. By definitions the separative and cooperative structures are sets of different component structures, while the sequential structures are sequences of component structures in which some component structures may appear more than once.
2. By definition a structure does not coincide with any of its components.

Thus, for examples, in this modelling $s = (a, a)$ and $s = [a, a]$ are not well-formed, while $r = \langle a, a \rangle$ is well-formed and is not $r_1 = \langle a \rangle$.

Example 1: $s = [a, \langle (b, c), d \rangle]$, $r = [b, \langle (a, c), a \rangle]$ are two structures of the same types $t(s) = t(r) = [*, \langle (*,*),* \rangle]$ and the type of $p = [c, \langle a, b \rangle]$ is $t(p) = [*, \langle *,* \rangle]$.

The structures in ***Definition*** 1 are defined in their linear forms. To each structure we can associate a tree graph T_s that is defined as below:

i. If $s = (a)$, $s = [a]$ or $s = \langle a \rangle$, then T_s is the tree graph described in Fig. 1, Fig. 2, and Fig. 3 respectively.

ii. If $s = (s_1, s_2, \dots, s_n)$, $s = [s_1, s_2, \dots, s_n]$ or $s = \langle s_1, s_2, \dots, s_n \rangle$, where $s_1, s_2, \dots, s_n$ are structures of A with trees $T_1, T_2, \dots, T_n$, respectively, then T_s is the tree described in Fig. 4, Fig. 5 and Fig. 6, respectively.

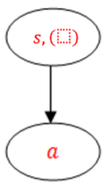

Fig. 1

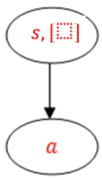

Fig. 2

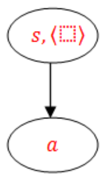

Fig. 3

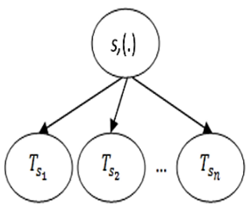

Fig. 4

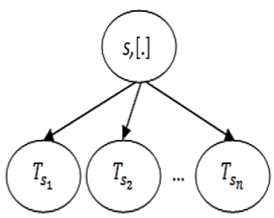

Fig. 5

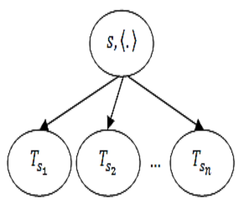

Fig. 6

The tree graph of the structure $s = [a, \langle (b, c), d \rangle]$ in Example 1 is given in Fig. 7. By the graph one can see that b and c operate as separate substructures of v, while a and u operate as cooperative substructures of s.

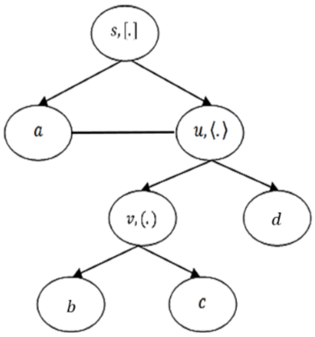

Fig. 7: The tree graph of the structure $s = [a,\ \langle (b,c), d\rangle]$

Special structures:

1. Substructures

For $s, r \in \boldsymbol{S_A}$ we say that s is a substructure of r, written as $s \leq r$, if $s \in \boldsymbol{SS}(\boldsymbol{r})$. It is easy to see that $\leq$ is a reflexive, transitive and anti symmetric relation, i.e. $\leq$ is a partial order on $\boldsymbol{S_A}$, therefore $\leq$ is a partial order on $\boldsymbol{SS}(s)$ for all $s \in \boldsymbol{S_A}$.

2. Independent substructures

For $s \in \boldsymbol{S_A}$we say that two substructures $s_1, s_2 \leq s$ are independent, if s_1, s_2 have no common substructures, i.e. $\boldsymbol{SS}(s_1) \cap \boldsymbol{SS}(s_2) = \emptyset$.

3. Separable structures

We say that $s \in \boldsymbol{S_A}$ is separable if there are pairwise independent substructures $s_1, s_2, \dots, s_n$ such that $s = (s_1, s_2, \dots, s_n)$.

4. Simple structures:

Simple structures are those structures where there are no two different substructures that are established from the same elements, i.e. for all $s_1, s_2 \in \boldsymbol{SS}(s)$, $\boldsymbol{AS}(s_1) = \boldsymbol{AS}(s_2)$ implies $s_1 = s_2$.

5. S-discrete structures:

S-discrete structures are those structures where only comparable substructures have common substructures, i.e. for all $s_1, s_2 \in \boldsymbol{SS}(s)$, if $\boldsymbol{SS}(s_1) \cap \boldsymbol{SS}(s_2) \neq \emptyset$ then $s_1 \leq s_2$ or $s_2 \leq s_1$.

6. A-discrete structures

E-discrete structures are those structures where only comparable substructures have common elements, i.e. for all $s_1, s_2 \in \boldsymbol{SS}(s)$, $\boldsymbol{AS}(s_1) \cap \boldsymbol{AS}(s_2) \neq \emptyset$ implies $s_1 \leq s_2$ or $s_2 \leq s_1$.

7. Separative, cooperative and sequential structures

$r \in \boldsymbol{S_A}$ is a separative, cooperative or sequential structure, if for all $s \leq r$, s is of the form $s = (s_1, s_2, \dots, s_n)$, $s = [s_1, s_2, \dots, s_n]$ or $s = \langle s_1, s_2, \dots, s_n\rangle$, respectively.

It is easy to see that every s-discrete structure is an e-discrete structure, but the converse is not always true.

Example 2:

	$[c, \langle a, b\rangle]$	$(\langle a, c\rangle, \langle a, b\rangle)$	$([c, \langle a, b\rangle], [\langle a, b\rangle, d])$	$[(a, b), \langle a, b\rangle]$
Simple structure	+	+	+	-
S-Discrete structure	+	-	-	-
A-Discrete structures	+	-	+	-

5. Models of organizations

In general an organization is a set of its sub-organizations (units):

$o = \{o_1, o_2, \dots, o_n\}$.

The organizations are studied with their features or characteristics suitably chosen for the aim of the study. For example, some frequent characteristics of organizations are discussed as in the following:

Organizations in functional modelling
An organization is a set of its sub-organizations that are arranged in a structure. First of all organizations are characterized by their functionality. In this study the organizations functionality is understood as a function mapping from some inputs to some outputs. The meaning of the functionality mapping should be explained accordingly to the aim of the research or the modelling. Thus an organization in the functional modelling is a pair

$\langle o, f\rangle$,

where o is the organizations identifier and $f: N^n \to N^m$ is its functional mapping that processes the inputs into the outputs.

Organizations in functionality-performance modelling
The performance of organizations is evaluated when they process inputs. If $\langle o, f\rangle$ is an organization where $f: N^n \to N^m$ is its functional mapping then the performance of o is defined also as a mapping $e: N^n \to N$. The performance of o in processing $(x_1, x_2, \dots, x_n)$ is $e(x_1, x_2, \dots, x_n)$.
An organization in the functionality-performance modelling should be denoted as a triple:

$\langle o, f, e\rangle$,

where $f: N^n \to N^m$ and $e: N^n \to N$ is the functional mapping and performance of the organization.

Organizations in material distribution modelling
Production scheduling and material distribution planning are important problems in logistics. Job shop scheduling was studied in [Bier]. Some problems of production scheduling were considered in [Gub]. Optimization of material distribution processes is always attractive topic for studies. When organizations are considered as material distributing units, the functional mapping should be considered as distribution function. The material distribution in fact is

targeted delivering, i.e. the items should be delivered to definite addresses. Moreover, the total of delivered items should not exceed the total of coming items and the remained items after distribution should not exceed its storage capacity. In other words the material distribution is completed only in the following conditions:

- If $f = (f_1, f_2, \dots, f_m)$ then for all $(x_1, x_2, \dots, x_n) \in N^n$

$$\sum_{i=1}^{m} f_i(x_1, x_2, \dots, x_n) \leq \sum_{i=1}^{n} x_i \qquad (1)$$

The condition (1) requires that the total of delivered items does not exceed the total of coming items.

- In the material distribution models an organizations storage capacity is bounded by a limit S. It is evident that an organization can process a series of inputs $X^p = \left(x_1^p, x_2^p, \dots, x_n^p\right), p = 1, \dots, k$ if

$$\sum_{p=1}^{k} \left(\sum_{i=1}^{n} x_i^p - \sum_{i=1}^{m} f_i\left(x_1^p, x_2^p, \dots, x_n^p\right) \right) \leq S \qquad (2)$$

By (2) we assume that an organization as a material distributing unit only operates well in processing a series of inputs, if the total of remained items after distribution does not exceed its storage capacity.

Thus an organization in the material distribution modelling should be denoted as a triple:

$\langle o, f, S \rangle$,

where $f: N^n \rightarrow N^m$ denotes the distributing function of the organization and $S \in N$ denotes the upper bound of its storage capacity.

Organizations in information delivering models

The organizations process information in different ways. The information delivering may be targeted or untargeted delivering. In targeted delivering the coming information should be processed by the recipient organization that in turn send the processed information to definite addresses. In untargeted delivering the processed information is shared to the all other related organizations. The requirement is that each organization is capable to process a set of information. In other words information delivering should satisfy the following conditions:

The functional mapping of information processing organizations is of the form:
$f: C \rightarrow I^m$,
where I is the set of all relevant information, and $C \subseteq I^n$ is the set of information that the organization can process.
The information processing organizations therefore should be denoted as a triple:

$\langle o, f, C \rangle$,

where $f: C \to I^m$ denotes the information processing function of the organization and $C \subseteq I^n$ denotes the set of information that the organization can process.

Organizations in multi-targeted models

In the above discussions we have proposed different models that are suitable for studies of structured organizations from different points of views. In fact the organizations can be examined in different structures at same time. Thus the organizations should be defined as a more complex systems with all features that are necessary for the studies.

For examples, when performance and capacity of material distribution are examined at the same time, organizations should be denoted as the systems of the form:

$\langle o, f_1, f_2, e, S \rangle$,

where $\langle o, f_2, S \rangle$ is the material distributing model, while $\langle o, f_1, e \rangle$ is the functionality-performance model of the organization.

6. Embedding of organizations into structures

Embedding of an organization $o = \{o_1, o_2, \dots, o_n\}$ into a structure $G = \langle V, E \rangle$ is the way we map all the units of the given organization o to the nodes of G. Then the edges of G denote the relationship between the sub-organizations. It is supposed that the embedding mapping is always unambiguous: The embedding mapping should be injective. Moreover, the embedding mapping should satisfy the following compliance condition:

For $o_i \in o$ let the functional mapping of o_i be $f^i: N^{n_i} \to N^{m_i}$. n_i and m_i denote the input and output index of o_i, respectively. Then we write also $in(o_i) = n_i$ and $out(o_i) = m_i$.

For $v \in V$ let
$IN(v) = \{v' \in V | [v', v] \in E\}$ be the set of higher-leveled nodes of v,
$OUT(v) = \{v' \in V | [v, v'] \in E\}$ be the set of lower-leveled nodes of v, and
$EQUAL(v) = \{v' \in V | [(v, v')] \in E\}$ be the set of equal-leveled nodes of v.

Then we denote also
$i(v) = |\{v' \in V | [v', v] \in E\}|$,
$o(v) = |\{v' \in V | [v, v'] \in E\}|$, and
$e(v) = |\{v' \in V | [(v, v')] \in E\}|$.

$i(v)$, $o(v)$ and $e(v)$ is called by incoming, outcoming and equivalent index of v, respectively.

Compliance condition:

In embedding of an organization into a structure the input index of sub-organizations must coincide with the incoming index of the associated nodes, and the output index of sub-organizations must coincide with the outcoming index of the associated node.

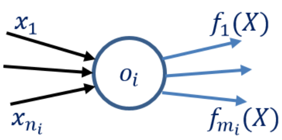

Fig. 8: The coincidence of input/output index of organization with the incoming/outcoming index of the node.

We can note that in more complex models the organizations may be embedded in the same time into different structures.

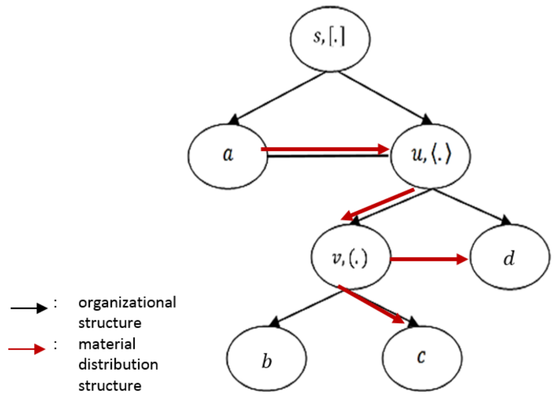

Fig. 9: The organization in different structures.

7. Problems with structured organizations

By modelling of structured organizations a lot of problems in different areas of management can be formulated in a formal setting. Some of these problems are purely theoretical, while the others are raised in practical analysis of companies. We consider below some of these problems:
Problems with structures and organizations: Modelling of structured organizations gives us separate views on structures and organizations. The relationships between structures and organizations should be analyzed in general and in special cases.

1) **Structurizing the organizations**: The question here is how can we structurize a given organizations. More exactly:
For an organization $o = \{o_1, o_2, \dots, o_n\}$ where $\langle o_i, f^i \rangle$ are sub-organizations with f^i functional mapping can we embed the given organization into a structure such that the compliance condition is satisfied?

2) **Special structurizing of the organizations**: The similar question can be put for special cases of organizations and structures:
For an organization with some characterizations and for a structure with some characterizations can we embed the given organization into a structure such that the compliance condition and all other condition of given organization and structure are satisfied?

Problems with functionality of organizations

1) **Determining and analyzing of functional capability of organizations**: In simplest models the structured organizations are sets of units with specified functions embedded into some structures. The question is: in general case can we determine and analyze the functional mapping of super organizations based on the functional mappings of their sub-organizations, if the rules of construction of super organizations are known?

2) **Functional capability of organizations**: The similar question is: What function can be executed by structured organizations?

The answer to the question would reveal the functional capability of structured organizations.

3) **Minimization of organizations of same functionality**: If a function can be executed by structured organizations, then can we determine the structured organizations that execute the given function with fewer sub-organizations?
Another version of this question: For a given organization can we determine the structured organizations that execute the same function with fewer sub-organizations?
The answer to the question may be a solution for the problem of reorganization.

4) **Composition and decomposition of organizations**: Can we compose a structured organization with some given characterizations from some given sub-organizations? Can we decompose a given structured organization into sub-organizations with some given characterizations?

Problems with functionality-performance models: In functionality-performance models of structured organizations we can see the dependency of the performance of super organizations on the performance of their sub-organizations. If these dependencies are known, the following questions are evident:

1) **Assessment of super organizations performance**: How can we appreciate the performance of super organizations and of the whole organization based on the performance of their sub-organizations?

2) **Optimization of organizations**: How can we raise the performance of the whole organization by reorganizing their sub-organizations and the whole organization?

Problems with material distribution models: The organizations in material distribution models play the role as receiver-supplier in a material distribution network. The two main constraints of these processes are: An organization can deliver less of items than the received quantity, and the storage capacity of the organization is bounded. The following problems can be considered for organizations in material distribution models, where by deadlock we understand the facts that some sub-organization cannot complete the distribution since the limitation of its storage capacity.

1) **Determination of deadlocks**: How can we determine those sub-organizations where in a given distribution process the distribution cannot be completed by the limitation of storage capacity?

More generally, the following problem is also an important problem in logistics:

2) **Determination of appropriate distribution process**: How can we determine those distribution processes that can be completed without any deadlocks?

Problems with information delivering models: Information processing and delivering is completed in different way. Here in information processing and delivering processes the information quantity is ignored. The only common characteristics of these processes is that the units are able to process and deliver only information in appropriate class. As in the case of material distribution models, the similar problems may be topics for further researches:

1) Determination of deadlocks: For a given source information how can we determine those sub-organizations that cannot manage the derived information in the process?

2) Determination of inappropriate information: How can we determine those source information whose derives in the process some sub-organizations cannot manage?

8. Conclusion

In this study an approach is sketched out to modelling of structured organizations. The essence of the approach is the structures, the organizations and the characteristics of organizations as different factors that jointly bring about the effectiveness of the whole organizations are studied separately. Formal definitions of these concepts are given, and based on the formal definitions a classification of structures is proposed. A model of structured organizations is a complex of operative units with their operational characteristics embedded into some structure. Several models of structured organizations are introduced with different characteristics suitable for description of these organizations in different processes. In the functional models the functionality of organizations is the target of study. In the functionality-performance models the performance of operative units should be main factor in optimization problems. Since the material distribution and information processing-delivering are completed in different ways, the structured organizations are studied with different characteristics in these models accordingly to the features of the material distribution and information delivering processes.
In this formal setting a series of problems concerning different models of structured organizations are set up. Some of these problems have theoretic significance. For examples, the question that which function can be executed by structured organizations is an interesting question for mathematics. Other questions are raised from practical areas of management, logistics and other fields of economics. More exactly, these questions focus on the reorganization, optimization, deadlock detection, etc. As we can see, the main result of the study is the proposal of an approach by which we can study separately the structure, the organizations and the characteristics of organizations. Though the questions posed in this study are not solved completely, the questions itself point out different directions for further studies.

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